Discrete Optimisation
Project 1: Lorry Dispatch

16th August 2016

A pharmaceutical company has a limited number of lorries to ship objects from their storehouse in Sweden to their distribution centre in Belgium. They have various kinds of objects, some of them with more specific shipping constraints that must be satisfied.

1 Statement

For this large shipping operation, the company wants to minimise the overall transportation costs. The path for the lorries being fixed, it comes from two main sources: the fact that a lorry is used (capital remuneration: fixed cost), and the weight it carries (fuel: variable cost).

Their operational research consultant proposes to follow these steps to solve this problem:

1. **Implement the basic model**, assigning objects to lorries with the length constraint, minimising the fixed cost of using lorries. Lorries have all the same cross section but may have different lengths. The objects have different lengths, but they are packed into standardised boxes whose width and height are the same as that of the lorries. The firm has a given number of available lorries; not all of them have to be used.

2. **Take into account weight constraints** and costs. Each lorry has a gross combination weight rating: the total mass of the cargo has physical and legal limits, depending on the lorry. Not respecting this constraint has very serious security impacts.

3. **Include the notion of refrigerator lorry**. Some objects must be kept at a low temperature to avoid breaking the cold chain and make some drugs unuseable. Some lorries can be equipped as refrigerator, but others cannot. Using such equipment dramatically increases the cost of transportation, as refrigeration is a rather energy-consuming process; it should be noted that standard objects can be transported in refrigerator lorries.

4. **Allow subcontractors**. To reduce the stress for the drivers and ensure all objects are transported in time, not all packages have to be transported by the company itself: it may decide to partly subcontract this work. The subcontractor imposes some cost for each object, based on its dimensions and weight, which is higher if the package must be kept refrigerated. Dependent objects cannot be handed over to the subcontractors.

5. **Consider dependencies**. Furthermore, some objects must be transported simultaneously, so that they are directly useable once in Belgium (such as vaccines and syringes). Those dependent objects must be carried in the same lorry: if an object a depends on an object b, then a must be shipped in the same lorry as object b.

6. **As a bonus step: allow teams**. Handling dependencies with forcing the items to be in the same lorry may be too conservative and prevent less costly solutions to be found. The dependency constraint is then replaced by team dependency. Lorries can be arranged in teams (whose size is limited) that stay together on the road: dependencies are achieved by transporting the objects within one team. If an object a depends on an object b, then a must be shipped in a lorry which belong to the same team as object b.

2 Available data

Symbol	Unit	Description
max_team_size	1	Maximum number of lorries in a team
lorries	1	Available number of lorries
max_weight[j]	tons	Maximum weight lorry j can carry
max_length[j]	metres	Maximum weight lorry j can carry
refrigerable[j]	1	True when lorry j can be equipped as refrigerator
fixed_cost[j]	€	Cost due to using lorry j
variable_cost[j]	€/ton	Cost due to lorry j carrying objects
refrigeration_cost[j]	€	Cost due to equipping lorry j as refrigerator
objects	1	Number of objects to carry
weight[i]	tons	Weight of object i
length[i]	metres	Length of object i
refrigeration_needed[i]	1	True when object i needs refrigeration
subcontractor_cost[i]	€	Cost due to object i being carried by the subcontractor
dependency_set ($\mathcal{D}$)	/	$(a, b) \in \mathcal{D}$ if objects a and b depend on each other
dependency_matrix (W)	1	W_{ab} is true if objects a and b depend on each other

Table 1: Available data.

3 Instructions

Individually, develop a sequence of mixed-integer linear models for this problem. Explain it in a **brief** report (including definition of variables, mathematical statement of the objective function and of the constraints, with brief explanations, following the steps defined in Section 1; no code shall be shown in the report; the best way of providing useful explanations is to use lists, and not large blocks of text) and implement it in Julia (one model per step), using the provided format for data inputs and solution outputs. Both the report (in PDF format) and your source code must be handed in on or before Sunday, 11th October 2015 on the submission platform, as a ZIP or 7Z archive.