Discrete Optimisation
Exercise Session 12: Constraint Programming

10th December 2015

Exercise 1 (sudoku)**.** Sudoku is a kind of puzzle game. It involves an $N \times N$ grid, some of them being filled. The goal is to fill all spaces with numbers between 1 and N^2 while meeting a series of constraints: "the same single integer may not appear twice in the same row, column or in any of the N $\sqrt{N} \times \sqrt{N}$ subregions of the $N \times N$ playing board" [Wikipedia].

1. Explain how a human could solve any sudoku puzzle using the same principles as constraint programming. Apply it on the given grid.
2. Write a constraint programming model to solve sudoku puzzles. Use only the constraint `alldifferent`.

1			
		2	
	3		4

Table 1: Example 2×2 sudoku grid.

Exercise 2 (magic square)**.** "A magic square is an arrangement of distinct integers in an $N \times N$ grid, where the numbers in each row, and in each column, and the numbers in the main and secondary diagonals, all add up to the same number" [Wikipedia]. This number is given by the following formula, based on the size of the grid N:

$$\frac{N\left(N^2+1\right)}{2}.$$

Thus, for a 3×3 grid, the three digits in each row, column, and diagonal must sum up to 15; for a 4×4 grid, they must sum up to 34; etc.

Write a constraint programming model to solve magic squares. Use only the constraint `alldifferent` and linear equalities.

Exercise 3 (n-queens)**.** The n-queens problem is about placing N queens in an $N \times N$ grid such that no two queens can threaten each other. The rules for queens are those of chess: a queen can attack another one if they are on the same row, column, or diagonal.

1. Write a constraint programming model to solve the n-queens problem. Use only the constraint `alldifferent`.
2. Write a mixed-integer linear program to solve the n-queens problem. Compare it to the constraint programming example. How many constraints does your model use? What is the domain of the variables?
3. Solve the problem for $N = 4$ using reasoning (domain filtering based on the constraints) and enumeration. How many solutions exist?