Discrete Optimisation
Exercise Session 1: Modelling

September 18, 2015

Exercise 1 (steel factory planning)**.** A steel factory manufactures spools; the director wants to plan the production for the next twelve weeks. The demand for spools is exactly known for each week. The cost for using the furnace can be divided into two parts: a fixed cost to get the furnace turned on and a variable cost proportional to the number of spools produced. Storing spools from a time period to another is costly, and is proportional to the number of spools to store.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Demand [10 spools]	7	5	3	5	5	9	1	8	5	6	2	2
Variable cost [1000€ per 10 spools]	2	2	2	2	2	2	2	2	2	2	2	2
Fixed cost [1000€]	16	16	16	16	16	16	16	16	16	16	16	16
Storage cost [1000€ per 10 spools]	1	1	1	1	1	1	1	1	1	1	1	1

Table 1: Spool factory planning requirements.

1. Formulate the problem of determining a production plan while minimizing the costs as a mixed-integer linear program.
2. Reformulate the planning problem when adding the following constraint: the blast-furnace must stay active at least three and at most four consecutive periods once started.

Exercise 2 (making groups)**.** 40 people participate in a game. For this purpose they must be divided in five groups of eight people. For each pair (i, j), a matrix has a value

- 0 if they don't know each other.
- 1 if they know each other a bit.
- 2 if they know each other well.

The goal is to create groups so that participants know as few other people as possible in each group. Formulate this problem by choosing a plausible objective function as a mixed-integer linear program.

Exercise 3 (nurse scheduling)**.** A working day in a hospital is subdivided in twelve periods of two hours. The staff requirements change from period to period. A nurse works eight hours a day and is entitled to a break after four hours.

Day	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24
Minimum nurses	15	14	14	17	22	20	10	20	18	15	20	20

Table 2: Nurses requirements for the hospital.

1. Determine the minimum number of nurses required to cover all requirements using a mixed-integer linear program.

2. If only 55 nurses are available, and assuming that it is not enough, the management allows for a certain number of nurses to work for a fifth period right after the last one. Determine a schedule the minimum number of nurses working overtime.

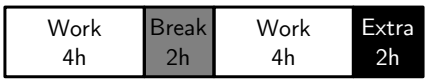

Figure 1: A nurse working day.

Exercise 4 (computer solutions)**.** Solve numerically the previous exercises by writing the model previously found with a modelling tool (such as JuMP, Pyomo, ZIMPL, etc.).