Discrete Optimisation
Exercise Session 7: Cuts

October 30, 2015

Exercise 1 (valid inequalities)**.** Find a valid inequality for the following sets:

1. $X = \{x \in \mathcal{B}, y \in \mathcal{B} \mid 12\,x - 6\,y \leq 7\}$.
2. $X = \{x \in \mathbb{R}^+, y \in \mathcal{B} \mid x \leq 20\,y, \quad x \leq 7\}$.
3. $X = \{x \in \mathbb{R}^+, y \in \mathbb{N} \mid x \leq 6\,y, \quad x \leq 16\}$.
4. $X = \left\{x \in \mathbb{R}^+, y \in \mathbb{N}^3 \mid -x - \frac{10}{3}y_1 + y_2 + \frac{11}{4}y_3 \leq \frac{21}{2}\right\}$. s
5. $X = \left\{(x, y, z) \in \mathcal{B}^3, s \in \mathbb{R}^+ \mid 2\,x + 3\,y + 9\,z - s \leq 32\right\}$.

Exercise 2 (cutting points)**.** Find a valid inequality for X which cuts the point x^*:

$$X = \left\{(v, w, x, y, z) \in \mathbb{Z}^5 \mid 9\,v + 12\,w + 8\,x + 17\,y + 13\,z \geq 50\right\},$$

$$x^* = \left(0, \frac{25}{6}, 0, 0, 0\right).$$

Exercise 3 (covers)**.** Find valid covers for the following X which cuts the points x^*:

1. $X = \left\{x \in \mathcal{B}^5 \mid 9\,v + 8\,w + 6\,x + 6\,y + 5\,z \leq 14\right\}$, $x^* = \left(0, \frac{5}{8}, \frac{3}{4}, \frac{3}{4}, 0\right)$.
2. $X = \left\{x \in \mathcal{B}^5 \mid 9\,v + 8\,w + 6\,x + 6\,y + 5\,z \leq 14\right\}$, $x^* = \left(\frac{1}{2}, \frac{1}{8}, \frac{3}{4}, \frac{3}{4}, 0\right)$.
3. $X = \left\{x \in \mathcal{B}^5 \mid 7\,v + 6\,w + 6\,x + 4\,y + 3\,z \leq 14\right\}$, $x^* = \left(\frac{1}{7}, 1, \frac{1}{2}, \frac{1}{4}, 1\right)$.
4. $X = \left\{x \in \mathcal{B}^5 \mid 12\,v - 9\,w + 8\,x + 6\,y - 3\,z \leq 2\right\}$, $x^* = \left(0, 0, \frac{1}{2}, \frac{1}{6}, 1\right)$.

Exercise 4 (lifting covers)**.** For the set $X = \left\{(u, v, w, x, y, z) \in \mathcal{B}^6 \mid 12\,u + 9\,v + 7\,w + 5\,x + 5\,y + 3\,z \leq 14\right\}$ and the cover $w + y + z \leq 2$:

1. Determine whether the cover is a facet of $X \cap \left\{(u, v, w, x, y, z) \in \mathcal{B}^6 \mid u = v = x = 0\right\}$.
2. Lift the inequality for X.