Discrete Optimisation
Exercise Session 2: Modelling

26th September 2016

After this session, you should be able to solve all exercises of Section 2.1 in the exercises book.

Exercise 1 (hospital placement — 2.1.1)**.** A country wants to reconsider its health system and seek for better ways of using the scarcer and scarcer resources available to the State. To this end, the government asks a consulting company where should the hospital be located based on the actual demand, if they could rebuild the complete system from scratch.

The demand is represented as a series of discrete points having some demand (for example, one point for each city, be it large or small). The hospital can only be built at some points (usually corresponding to the large cities). Meeting the demand has some cost that depends on the city where the hospital is located (the largest part coming from ambulances, which must travel a long way before rescuing people; casualties due to the emergency services arriving too late are already considered in these costs); these costs are given in a matrix (one dimension corresponds to the demand points, the other to the potential hospital). Opening a hospital can only be done at a high price.

More precisely, hospitals can be located at three places, with the costs $\begin{pmatrix}500 & 800 & 1800\end{pmatrix}$. The demand is modelled as six points: $\begin{pmatrix}24 & 12 & 8 & 96 & 1 & 48\end{pmatrix}$. The cost matrix of serving the demand from the three possible hospitals is:

$$\begin{pmatrix} 1 & 20 & 10 & 5 & 50 & 9 \\ 18 & 80 & 11 & 4 & 40 & 2 \\ 20 & 25 & 30 & 1 & 35 & 10 \end{pmatrix}.$$

The government goal is to minimise their total costs (i.e. building and operating the hospitals). Model the situation as a mixed-integer linear program.

Exercise 2 (nurse scheduling — 2.1.2)**.** A working day in a hospital is subdivided in twelve periods of two hours. The staff requirements change from period to period. A nurse works eight hours a day and is entitled to a break after four hours.

Day	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24
Minimum nurses	15	14	14	17	22	20	10	20	18	15	20	20

Table 1: Nurses requirements for the hospital.

1. Determine the minimum number of nurses required to cover all requirements using a mixed-integer linear program.

2. If only 55 nurses are available (which it is not enough for the given data), the management allows for a certain number of nurses to work for a fifth period right after the last one. Determine a schedule the minimum number of nurses working overtime.

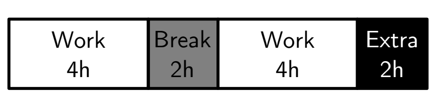

Figure 1: A nurse working day.

Exercise 3 (team making — 2.1.3)**.** 40 people participate in a game. For this purpose they must be divided in five groups of eight people. For each pair (i, j), a matrix has a value

- 0 if they don't know each other.
- 1 if they know each other a bit.
- 2 if they know each other well.

The goal is to create groups so that participants know as few other people as possible in each group. Formulate this problem by choosing a plausible objective function as a mixed-integer linear program.

Exercise 4 (steel mill planning — 2.1.4)**.** A steel mill manufactures I-beams; the director wants to plan the production for the next twelve weeks. The demand for I-beams is exactly known for each week. The cost for using the furnace can be divided into two parts: a fixed cost to get the furnace turned on and a variable cost proportional to the number of beams produced. Storing beams from a time period to another is costly, and is proportional to the number of beams to store.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Demand [10 beams]	7	5	3	5	5	9	1	8	5	6	2	2
Variable cost [1000€ per 10 beams]	2	2	2	2	2	2	2	2	2	2	2	2
Fixed cost [1000€]	16	16	16	16	16	16	16	16	16	16	16	16
Storage cost [1000€ per 10 beams]	1	1	1	1	1	1	1	1	1	1	1	1

Table 2: I-beams factory planning requirements.

1. Formulate the problem of determining a production plan while minimizing the costs as a mixed-integer linear program.
2. Reformulate the planning problem when adding the following constraint: the blast-furnace must stay active at least three and at most four consecutive periods once started.

Exercise 5 (ground meat preparation — 2.1.5)**.** In the food-processing industry, ground-meat preparation is a basic step for many recipes (from the most basic ones, like sausages or hamburger patties, to more complex ones, such as keema or lasagne). Its quality is dictated by the quantity of fat.

To prepare this ground meat, industrial butchers prepare trays with pieces of meat, after the slaughterhouse and the retrieval of the best pieces of meat. These trays have some quantity of meat, of which a certain proportion of fat.

For the rest of the food-processing supply chain, some quantity of ground meat must be produced by assembling those trays (it is not possible to split a tray, e.g. in two and just use half of the total tray). As all trays do not have predictable weight or fat content, the butchers have a tolerance of 5% with the weight (i.e. they may assemble trays whose total weight is at most 5% less or more than what required). Their goal is to be as close as possible to the requested quantity of fat for one recipe at a time by selecting a subset of the available trays (there may be more or less fat than requested).

Formulate this problem as a mixed-integer linear program.

Exercise 6 (computer solutions)**.** Solve numerically the previous exercises by writing the model previously found with a modelling tool (such as JuMP, Convex.jl, Pyomo, ZIMPL, CVX, etc.).