Discrete Optimisation
Exercise Session 4: Branch-and-bound algorithm

14th October 2016

After this session, you should be able to solve all exercises of Chapter 3 in the exercises book.

Exercise 1 (geometric solving — 3.0.1 (§2))**.** Solve the following problem geometrically using the branch-and-bound algorithm (i.e. by drawing the feasible area, finding the extreme points by hand—as the simplex algorithm would).

$$\begin{array}{ll} \max & 2\,x + 3\,y \\ \text{s.t.} & -\frac{2}{3}\,x + y \leq \frac{5}{2} \\ & \frac{1}{3}\,x + y \leq \frac{9}{2} \\ & 2\,x + y \leq 14 \\ & (x, y) \in \mathbb{Z}_+^2. \end{array}$$

Exercise 2 (tree construction — 3.0.2)**.**

1. Give the tightest lower and upper bounds on the optimal value for the objective based on the tree of Figure 1. Compute the gap.
2. With the given partial tree, give the nodes which should be pruned (and why), and which could be explored further.
3. Explore Gurobi's MIP log and link it to the branch-and-bound tree.

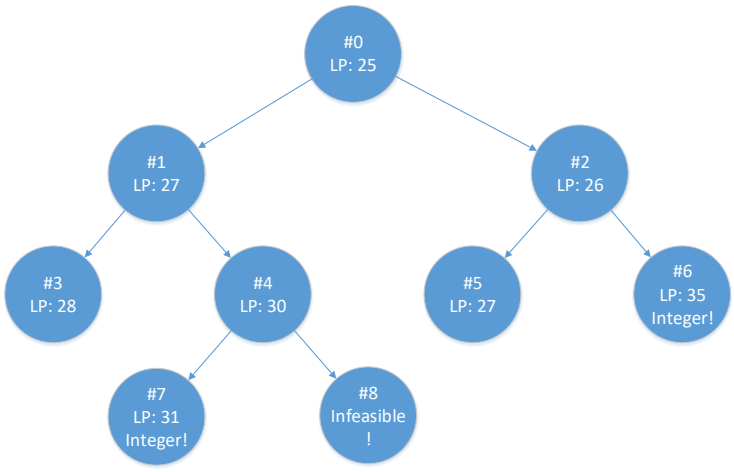

Figure 1: A partial branch-and-bound tree.

Exercise 3 (worst case — 3.0.3)**.** Give an example where the branch-and-bound algorithm performs a large number of iterations (when compared to the number of variables) to find a first integer solution.