Discrete Optimisation
Exercise Session 9: Cuts

3rd January 2017

After this session, you should be able to solve all exercises of Chapter 5 in the exercises book.

Exercise 1 (lifting covers)**.** For the set $X = \{(u, v, w, x, y, z) \in \mathcal{B}^6 \mid 12\,u + 9\,v + 7\,w + 5\,x + 5\,y + 3\,z \leq 14\}$ and the cover $w + y + z \leq 2$:

1. Determine whether the cover is a facet of $X \cap \{(u, v, w, x, y, z) \in \mathcal{B}^6 \mid u = v = x = 0\}$.
2. Lift the inequality for X.

Exercise 2 (optimal subtour elimination generation)**.** The generalised subtour elimination constraint can be used to formulate the prize-collecting travelling salesman problem: with respect to the travelling salesman problem, travelling through an edge e has a cost c_e, and visiting a city j allows them to make a profit of f_j; not all cities have to be visited, but the salesman still must follow a cycle, starting at the first city.

1. Give a MILP model for the prize-collecting travelling salesman problem using the generalised subtour elimination constraint (GSEC).
2. What could be practical problems of implementing this formulation?
3. Derive a separation procedure that could be used for a lazy-constraint-based implementation of the problem. The generated constraint must maximise the violation.
4. Solve the following instance of the prize-collecting travelling salesman problem with lazy constraint generation.

$$c_e = \begin{pmatrix} 0 & 4 & 3 & 3 & 5 & 2 & 5 \\ 4 & 0 & 5 & 3 & 3 & 4 & 7 \\ 3 & 5 & 0 & 4 & 6 & 0 & 4 \\ 3 & 3 & 4 & 0 & 4 & 4 & 6 \\ 5 & 3 & 6 & 4 & 0 & 5 & 8 \\ 2 & 4 & 0 & 4 & 5 & 0 & 3 \\ 5 & 7 & 4 & 6 & 8 & 3 & 0 \end{pmatrix},$$

$$f_j = \begin{pmatrix} 2 & 4 & 1 & 3 & 7 & 1 & 7 \end{pmatrix}.$$